ST. VIVEKANAND PUBLIC SCHOOL, SADABAD

CONTINUITY AND DIFFERENTIABILITY (CONTINUITY)

Class 12 - Mathematics

Section A

1. Find the value of f(0), so that the function $f(x)=\frac{\sqrt{a^2-ax+x^2}-\sqrt{a^2+ax+x^2}}{\sqrt{a+x}-\sqrt{a-x}}$ becomes continuous for all x, given by **[1]**

a) $a^{1/2}$
b) $-a^{1/2}$
c) $a^{3/2}$
d) $-a^{3/2}$

2. The function $f(x)=\begin{cases} x^2a & , \quad 0\le x<1 \\ a & , \quad 1\le x<\sqrt{2} \\ \frac{2b^2-4b}{x^2} & , \quad \sqrt{2}\le x<\infty \end{cases}$ is continuous for $0\le x<\infty$, then the most suitable values of a and b are **[1]**

a) a = -1, b = 1
b) a = -1, b = 1^+
c) a = - 1, b = -1
d) none of these

3. Let $f(x)=\begin{cases} \frac{1}{|x|} & \text{for } |x|\ge 1 \\ ax^2+b & \text{for } |x|<1 \end{cases}$ If f(x) is continuous and differentiable at any point, then **[1]**

a) a = 1, b = –1
b) $a=\frac{1}{2}, b=-\frac{3}{2}$
c) $a=\frac{1}{2}, b=\frac{3}{2}$
d) none of these

4. The function f(x) = cot x is discontinuous on the set **[1]**

a) $\left\{x=(2n+1)\frac{\pi}{2}; n\in\mathbf{Z}\right\}$
b) $\{x=2n\pi : n\in\mathbf{Z}\}$
c) $\left\{x=\frac{n\pi}{2}; n\in\mathbf{Z}\right\}$
d) $\{x=n\pi : n\in\mathbf{Z}\}$

5. Find the value of f(0), so that the function $f(x)=\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}\ (x\ne 0)$ is continuous, is given by **[1]**

a) 6
b) $\frac{2}{3}$
c) 4
d) 2

6. If $f(x)=\begin{cases} \frac{\sin(\cos x)-\cos x}{(\pi-2x)^2} & , x\ne\frac{\pi}{2} \\ k & , x=\frac{\pi}{2} \end{cases}$ is continuous at $x=\frac{\pi}{2}$, then k is equal to **[1]**

a) 1
b) -1
c) 0
d) $\frac{1}{2}$

7. The function f(x) = $\begin{cases} 1+x, & \text{when } x\le 2 \\ 5-x, & \text{when } x>2 \end{cases}$ is continuous and differentiable at x=2 ,yes or no **[1]**

a) Differentiable but not continuous at x = 2
b) Continuous as well as differentiable at x = 2

c) Continuous but not differentiable at x = 2

d) None of these

8. If $f(x)=\begin{cases}\frac{\log(1+ax)-\log(1-bx)}{x} & ,x\neq 0\\ k & ,x=0\end{cases}$ and f(x) is continuous at x = 0, then the value of k is **[1]**

a) a - b

b) a + b

c) log a + log b

d) none of these

9. If $f(x)=\begin{cases}mx+1, & \text{if } x\leq\frac{\pi}{2}\\ \sin x+n, & \text{if } x>\frac{\pi}{2}\end{cases}$ is continuous at $x=\frac{\pi}{2}$ then **[1]**

a) $m=n=\frac{\pi}{2}$

b) $n=\frac{m\pi}{2}$

c) m = 1, n = 0

d) $m=\frac{n\pi}{2}+1$

10. The function $f(x)=\frac{\sin(\pi[x-\pi])}{4+[x]^2}$,where [.] denotes the greatest integer function, is **[1]**

a) differentiable for all x but not continuous at some x.

b) none of these

c) continuous for all x but not differentiable at some x

d) continuous as well as differentiable for all x ∈ R

11. The function f(x) = |cos x| is **[1]**

a) everywhere continuous but not differentiable at (2n + 1) $\frac{\pi}{2}$, n ∈ Z

b) everywhere continuous and differentiable

c) neither continuous nor differentiable at (2n + 1) $\frac{\pi}{2}$, n ∈ Z

d) none of these

12. The function $f(x)=\frac{4-x^2}{4x-x^3}$ is **[1]**

a) none of these

b) discontinuous at only one point

c) discontinuous at exactly two points

d) discontinuous at exactly three points

13. Determine the value of the constant k so that the function $f(x)=\begin{cases}\frac{x^2-3x+2}{x-1} & ,\text{ if } x\neq 1\\ k & ,\text{ if } x=1\end{cases}$ is continuous at x = 1 **[1]**

14. Determine the value of 'k' for which the following function is continuous at x = 3 : f(x) = $\begin{cases}\frac{(x+3)^2-36}{x-3}, x\neq 3\\ k \quad ,x=3\end{cases}$. **[1]**

15. Discuss the continuity of the function f(x) given by(x) = $\begin{cases}2-x, & x<2\\ 2+x, & x\geq 2\end{cases}$ at x = 2. **[1]**

16. Discuss the continuity of the function f defined by f (x) = $\frac{1}{x}, x\neq 0$ **[1]**

17. Discuss the continuity of $f(x)=\begin{cases}2x-1, & x<0\\ 2x+1, & x\geq 0\end{cases}$ at x = 0. **[1]**

18. Determine f(0) so that the function f(x) defined by f(x) = $\frac{(4^x-1)^3}{\sin\frac{x}{4}\log\left(1+\frac{x^2}{3}\right)}$ becomes continuous at x = 0. **[1]**

19. Show that the function f defined by $f(x)=\begin{cases}x\sin\frac{1}{x}, x\neq 0\\ 0, x=0\end{cases}$ is continuous at x = 0. **[1]**

20. Show that f(x) = x^3 is continuous at x = 2. **[1]**

21. Examine the function for continuity. **[1]**

$f(x)=\frac{1}{x-5}$, x ≠ 5

22. If the function $f(x)=\frac{\sin 10x}{x}, x\neq 0$ is continuous at x = 0, find f(0). **[1]**

Section B

23. Find the points of discontinuity, if any, of the function: $f(x)=\begin{cases}\frac{x^4+x^3+2x^2}{\tan^{-1}x} & \text{, if } x\neq 0\\ 10 & \text{, if } x=0\end{cases}$ **[2]**

24. Find the value of k so that the function f is continuous at the indicated point: $f(x)=\begin{cases}3x-8, & if\ x\leqslant 5\\ 2k, & if\ x>5\end{cases}$ at x = 5. **[2]**

25. Prove that the function defined by f(x) = tan x is a continuous function. **[2]**

26. Find the value of k for which $f(x)=\begin{cases}kx+5, & \text{when } x\leq 2\\ x-1, & \text{when } x>2\end{cases}$ is continuous at x = 2. **[2]**

27. Show that sec x is a continuous function. **[2]**

28. Show that the function f(x) = 2x - |x| is continuous at x = 0. **[2]**

29. Discuss the continuity of the function $f(x)=\begin{cases}\frac{|x|}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ **[2]**

30. Discuss the continuity of the following function at x = 0 **[2]**

$$f(x)=\begin{cases}\dfrac{x^4+2x^3+x^2}{\tan^{-1}x}, & x\neq 0\\ 0, & x=0\end{cases}$$

31. Find the value of k for which the function $f(x)=\begin{cases}\frac{x^2+3x-10}{x-2}, & x\neq 2\\ k & ,x=2\end{cases}$ is continuous at x = 2 **[2]**

32. Show that the function f(x)= $\begin{cases}\frac{x}{|x|}, & \text{when } x\neq 0\\ 1, & \text{when } x=0\end{cases}$. **[2]**

Section C

33. Discuss the continuity of the function f, where f is defined by: $[f(x)=\begin{cases}-2, & \text{if } x\leq -1\\ 2x, & \text{if } -1<x\leq 1\\ 2, & \text{if } x>1\end{cases}]$ **[3]**

34. Find the relationship between a and b, so that the function f defined by f(x)= $\begin{cases}ax+1, & \text{if } x\leq 3\\ bx+3, & \text{if } x>3\end{cases}$ is continuous at x = 3. **[3]**

35. Find all points of discontinuity of f where f is defined as follows,f(x) = $\begin{cases}|x|+3, & x\leq -3\\ -2x, & -3<x<3\\ 6x+2, & x\geq 3\end{cases}$ **[3]**

36. Discuss the continuity of the function f, where f is defined by: $f(x)=\begin{cases}2x, & \text{if } x<0\\ 0, & \text{if } 0\leq x\leq 1\\ 4x, & \text{if } x>1\end{cases}$ **[3]**

37. For what values of λ is the function f(x) = $\begin{cases}\lambda\left(x^2-2x\right), & \text{if } x\leq 0\\ 4x+1, & \text{if } x>0\end{cases}$ is continuous at x = 0? **[3]**

38. Find all points of discontinuity of f, where f is defined by: $f(x)=\begin{cases}|x|+3, & if\quad x\leq -3\\ -2x, & if\quad -3<x<3\\ 6x+2 & if\quad x\geq 3\end{cases}$ **[3]**

39. Find the value of k so that the function f is continuous at the indicated point: **[3]**

$f(x)=\begin{cases}Kx+1\ if\,x\leq\pi\\ \cos x\ if\ x>\pi\end{cases}$ at $x=\pi$

40. Find the values of a and b so that the function f given by $f(x)=\begin{cases}1, & \text{if } x\leq 3\\ ax+b, & \text{if } 3<x<5\\ 7, & \text{if } x\geq 5\end{cases}$ is continuous at x = 3 and x = 5 **[3]**

Section D

41. **Read the text carefully and answer the questions:** **[5]**

The function f(x) will be discontinuous at x = a if f(x) has

- Discontinuity of first kind : $\lim\limits_{h \to 0} f(a-h)$ and $\lim\limits_{h \to 0} f(a+h)$ both exist but are not equal. If is also known as irremovable discontinuity.
- Discontinuity of second kind : If none of the limits $\lim\limits_{h \to 0} f(a-h)$ and $\lim\limits_{h \to 0} f(a+h)$ exist.
- Removable discontinuity: $\lim\limits_{h \to 0} f(a-h)$ and $\lim\limits_{h \to 0} f(a+h)$ both exist and equal but not equal to f(a).

(i) $f(x)=\begin{cases}\frac{x^2-9}{x-3}, & \text{for } x \neq 3 \\ 4, & \text{for } x=3\end{cases}$, then at x = 3

a) none of these
b) f has irremovable discontinuity
c) f has removable discontinuity
d) f is continuous

(ii) Let f(x) = $\begin{cases}x+2, & \text{if } x \leq 4 \\ x+4, & \text{if } x>4\end{cases}$ then at x = 4

a) f has irremovable discontinuity
b) none of these
c) f has removable discontinuity
d) f is continuous

(iii) Consider the function f(x) defined as f(x) = $\begin{cases}\frac{x^2-4}{x-2}, & \text{for } x \neq 2 \\ 5, & \text{for } x=2\end{cases}$, then at x = 2.

a) f is continuous if f(2) = 3
b) f has irremovable discontinuity
c) f has removable discontinuity
d) f is continuous

(iv) If f(x) = $\begin{cases}\frac{x-|x|}{x}, & x \neq 0 \\ 2, & x=0\end{cases}$, then at x = 0.

a) f has irremovable discontinuity
b) f is continuous
c) f has removable discontinuity
d) none of these

(v) If f(x) = $\begin{cases}\frac{e^x-1}{\log(1+2x)}, & \text{if } x \neq 0 \\ 7, & \text{if } x=0\end{cases}$, then at x = 0.

a) f is continuous if f(0) = 2
b) f has removable discontinuity
c) f is continuous
d) f has irremovable discontinuity

42. **Read the text carefully and answer the questions:** **[5]**

- A function f(x) is said to be continuous in an open interval (a, b) if it is continuous at every point in this interval.
- A function f(x) is said to be continuous in the closed interval [a, b], if f(x) is continuous in (a, b) and $\lim\limits_{h \to 0} f(a+h)=f(a)$ and $\lim\limits_{h \to 0} f(b-h)=f(b)$

If function f(x) = $\begin{cases}\frac{\sin(a+1)x+\sin x}{x} & , x<0 \\ c & , x=0 \\ \frac{\sqrt{x+bx^2}-\sqrt{x}}{bx^{3/2}} & , x>0\end{cases}$ is continuous at x = 0, then

(i) The value of a is

a) $\frac{1}{2}$
b) 0
c) $\frac{-1}{2}$
d) $\frac{-3}{2}$

(ii) The value of b is

a) -1

b) any real number

c) 1

d) 0

(iii) The value of c is

a) -2

b) -1

c) $\frac{1}{2}$

d) 1

(iv) The value of a + c is

a) -1

b) -2

c) 0

d) 1

(v) The value of c - a is

a) 2

b) -1

c) 1

d) 2

43. Show that the function $f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0, x = 0 \end{cases}$ is continuous but not differentiable at x = 0, if 0 < m < 1. **[5]**

44. Discuss the continuity and differentiability $f(x) = \begin{cases} 1 - x & , \quad x < 1 \\ (1-x)(2-x), & 1 \leq x \leq 2 \\ 3 - x & , \quad x > 2 \end{cases}$ **[5]**

45. Discuss the continuity of the function **[5]**

$$f(x) = \begin{cases} -2 & if \quad x \leqslant -1 \\ 2x, & if \quad -1 < x \leqslant 1 \\ 2 & if \quad x > 1 \end{cases}$$

46. Find all the points of discontinuity of f defined by f(x) = |x| – |x + 1|. **[5]**

47. Find the values of p and q for which $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ p, & \text{if } x = \frac{\pi}{2} \\ \frac{q(1-\sin x)}{(\pi - 2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$ is continuous at x = $\frac{\pi}{2}$. **[5]**

48. The function f(x) will be discontinuous at x = a if f(x) has **[5]**

- Discontinuity of first kind : $\lim_{h \to 0} f(a-h)$ and $\lim_{h \to 0} f(a+h)$ both exist but are not equal. If is also known as irremovable discontinuity.
- Discontinuity of second kind : If none of the limits $\lim_{h \to 0} f(a-h)$ and $\lim_{h \to 0} f(a+h)$ exist.
- Removable discontinuity: $\lim_{h \to 0} f(a-h)$ and $\lim_{h \to 0} f(a+h)$ both exist and equal but not equal to f(a).

Based on the above information, answer the following questions.

i. $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & \text{for } x \neq 3 \\ 4, & \text{for } x = 3 \end{cases}$, then at x = 3

a. f has removable discontinuity

b. f is continuous

c. f has irremovable discontinuity

d. none of these

ii. Let f(x) = $\begin{cases} x+2, & \text{if } x \leq 4 \\ x+4, & \text{if } x > 4 \end{cases}$ then at x = 4

a. f is continuous

b. f has removable discontinuity

c. f has irremovable discontinuity

d. none of these

iii. Consider the function f(x) defined as f(x) = $\begin{cases} \frac{x^2-4}{x-2}, & \text{for } x \neq 2 \\ 5, & \text{for } x = 2 \end{cases}$, then at x = 2

a. f has removable discontinuity

b. f has irremovable discontinuity

c. f is continuous

d. /is continuous if f(2) = 3

iv. If f(x) = $\begin{cases} \frac{x-|x|}{x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$, then at x = 0

a. f is continuous

b. f has removable discontinuity

c. f has irremovable discontinuity

d. none of these

v. If f(x) = $\begin{cases} \frac{e^x-1}{\log(1+2x)}, & \text{if } x \neq 0 \\ 7, & \text{if } x = 0 \end{cases}$, then at x = 0

a. f is continuous if f(0) = 2

b. f is continuous

c. f has irremovable discontinuity

d. f has removable discontinuity