ST. VIVEKANAND PUBLIC SCHOOL, SADABAD

HOLIDAY HOMEWORK

Class 12 - Mathematics

Section A

1. Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$. **[3]**
2. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$. **[3]**

Section B

3. Prove that: $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$ **[4]**

Section C

Question No. 4 to 8 are based on the given text. Read the text carefully and answer the questions: **[5]**

A relation R on a set A is said to be an equivalence relation on A iff it is

- Reflexive i.e., (a, a) ∈ R ∀ a ∈ A.
- Symmetric i.e., (a, b) ∈ R ⇒ (b, a) ∈ R ∀ a, b ∈ A.
- Transitive i.e., (a, b) ∈ R and (b, c) ∈ R ⇒ {a, c) ∈ R ∀ a,b, c ∈ A.

4. If the relation R = {(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)} defined on the set A - {1, 2, 3}, then R is

   a) reflexive b) symmetric

   c) equivalence d) transitive

5. If the relation R = {(1, 2), (2, 1), (1, 3), (3, 1)} defined on the set A = {1, 2, 3}, then R is

   a) reflexive b) transitive

   c) equivalence d) symmetric

6. If the relation R on the set N of all natural numbers defined as R - {(x, y) : y = x + 5 and x < 4}, then R is

   a) reflexive b) equivalence

   c) symmetric d) transitive

7. If the relation R on the set A = {1, 2, 3, ... 13, 14} defined as R = {(x, y) : 3x - y = 0}, then R is

   a) none of these b) symmetric

   c) transitive d) reflexive

8. If the relation R on the set A = {1, 2, 3} defined as R = {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}, then R is

   a) transitive only b) symmetric only

   c) equivalence d) reflexive only

9. Show that the function f : R → {x ∈ R : -1 < x < 1} defined by $f(x) = \frac{x}{1+|x|}$, x ∈ R is one-one and onto **[5]**

function.

10. Let A = R – {3} and B = R – {1}. Consider the function f: A ⇒B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

11. Show that the function f : R $\rightarrow$ R defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in R$, is neither one-one nor onto. **[5]**

12. Find X and Y, if $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ **[5]**

13. Three shopkeepers A, B and C go to a store to buy stationery. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs ₹1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill. **[5]**

14. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1} A^{-1}$ **[5]**

15. If $f(x) = ax^2 + bx + c$ is a quadratic function such that f(1) = 8, f(2) = 11 and f(-3) = 6, find f(x) by using determinants. Also, find f(0). **[5]**

16. Verify A (adj. A) = (adj. A) A = |A|I: **[5]**

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

17. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, find BA and use this to solve the system of equations y + 2z = 7, x - y = 3, 2x + 3y + 4z = 17. **[5]**